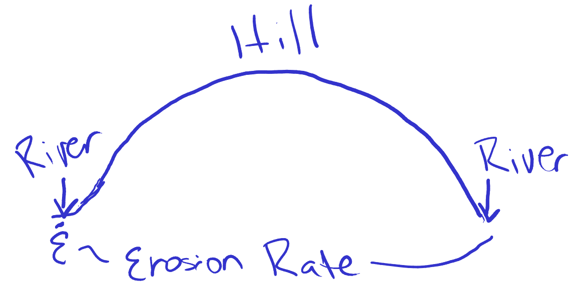
Why are hilltops round?

↳ Hilltops are <u>Parabolic</u>



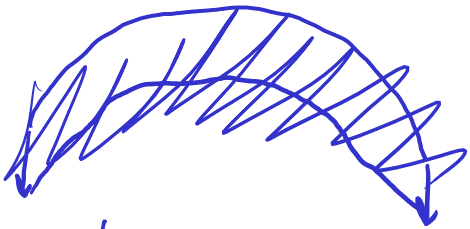
$$\frac{\partial z}{\partial t} = -k_m \frac{\partial^2 z}{\partial x^2} = \dot{\varepsilon}$$

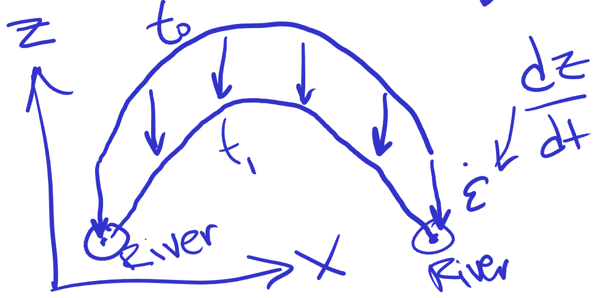
PDE

partial

↑ Diffusivity

Motion of hillslope mobile material





ODE

(ordinary differential equation)

$$\dot{\varepsilon} = -k_m \frac{d^2 z}{dx^2}$$

z : position (elevation)

$\frac{dz}{dx}$: slope

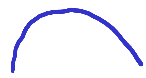
$\frac{d^2 z}{dx^2}$: curvature

$\boxed{z(x) : \text{GOAL HERE}}$

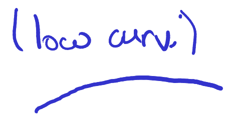
$$\frac{\dot{\varepsilon}}{-k_m} = \frac{d^2 z}{dx^2} = \gamma$$

↑ convenience less writing

Erosion Rate ↑ : Curvature ↑



Hillslope diffusivity↑ (erodibility) : Curvature ↓



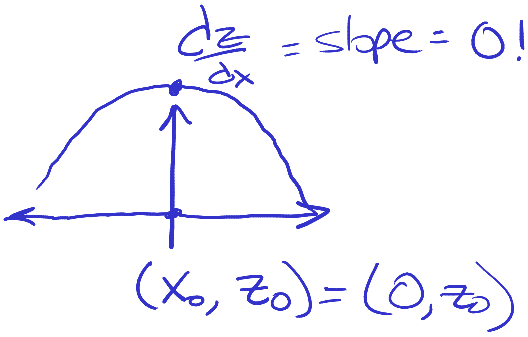
$$\frac{d^2 z}{dx^2} = \gamma$$

SEPARATION OF VARIABLES.

$$\frac{d^2z}{dx^2} = \gamma = \frac{d}{dx}\left(\frac{dz}{dx}\right)$$

$$\int \cancel{d}\left(\frac{dz}{dx}\right) = \int \gamma \, dx = \gamma x + C_o$$

$$\frac{dz}{dx} = \gamma x + \underline{C_o}$$



$$0 = \gamma 0 + \underbrace{C_o}_{=0}.$$

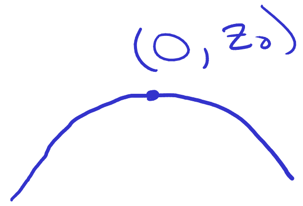
$$\frac{dz}{dx} = \gamma x = -\frac{\dot{\varepsilon}}{k_m}$$

$\dot{\varepsilon}\uparrow$, steeper hill

$k_m\uparrow$, gentler hillslope

Separation of variables (again)

$$\int dz = \int \gamma x \, dx$$



$$z = \gamma \frac{x^2}{2} + C_o$$

$$z_o = \cancel{\gamma \frac{0^2}{2}}^{\to 0} + C_o \;;\quad z_o = C_o.$$

$\dot{\varepsilon} > 0.$

$$z = z_o + \frac{\gamma}{2}x^2 \qquad \boxed{z = z_o - \frac{\dot{\varepsilon}}{2K_m}x^2}$$

Parabolic hilltops

Hilltop curvature