STREAM POWER & BEDROCK EROSION

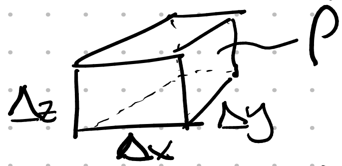
Energy

$PE = m\,g\,h \equiv z$

mass — gravitational acceleration — elevation

not

density



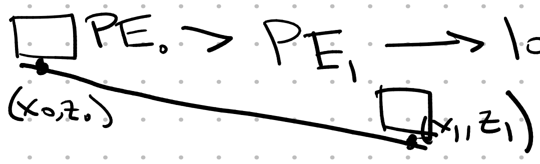
Control volume $\forall$

Assuming $h \leftarrow$ thickness of flow

$h \ll z$

$PE = \rho \forall g z$

elevation of the channel bed



$PE_0 > PE_1 \longrightarrow$ loss of PE downstream

$\downarrow$

converted..

Heat: shear $\rightarrow$ turbulence

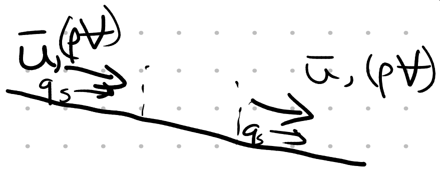
"shear heating"

Kinetic Energy: ~~flow velocity~~ (a) — steady ~~uniform~~ flow

~~sediment transport~~ (b) $\rightarrow q_s \propto \tau_b^{n?}$

bedrock erosion (c)

(a) $KE = \frac{1}{2} m \bar{u}^2$



Before: $m = 0$

$\bar{u}_{br\,(eroded)} = 0$

What is the rate of bedrock erosion?
What is the rate of PE → KE conversion?

Stream Power

Rate of PE loss in a channel cross section per unit time

Cross section:   Δx

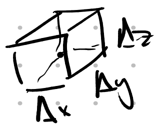
$$PE = \underbrace{\rho \forall}_{m} g z$$

$$\Omega = \frac{-\Delta PE}{\Delta x \Delta t} = -\frac{\rho \forall g}{\Delta x}\frac{\Delta z}{\Delta t}$$

stream power

interested in conversion into KE, heat

← Why did I separate this equation this way

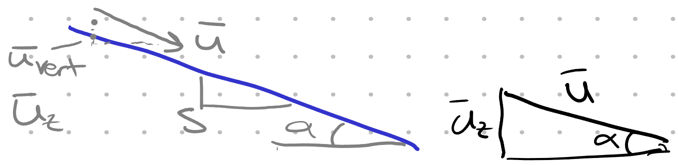
$$\forall = \Delta x \Delta y \Delta z$$ 

Control volume.

$$= -\frac{\rho \cancel{\Delta x} \Delta y \Delta z\, g}{\cancel{\Delta x}}\left(\frac{\Delta z}{\Delta t}\right)$$

← What is rate of drop of water elevation?

$$\frac{\text{vertical distance}}{\text{time}}$$



$$\sin(\alpha) = \frac{\bar{u}_z}{\bar{u}}$$

Small angle formula

if α is small.

then $\cos(\alpha) \simeq 1$

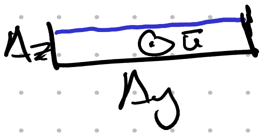
$\sin(\alpha) \approx \tan(\alpha) = S$

$$S = \frac{\bar{u}_z}{\bar{u}} \longrightarrow \therefore \; \bar{u}_z = \bar{u} S = \frac{\Delta z}{\Delta t}$$

$$\Omega = \rho \, \Delta y \, \Delta z \, g \left(\frac{\Delta z}{\Delta t} \right)$$

$$\Omega = \rho \, \Delta y \, \Delta z \, g \, \bar{u} S$$

Rectangular channel approximation



$Q = \Delta y \, \Delta z \, \bar{u}$

$\uparrow$ discharge

$$\boxed{\Omega = \rho g Q S}$$

Stream power

(steady, uniform flow)

$\left(\boxed{\tau_b = \rho g h S} \right)$

$$\omega = \frac{\Omega}{b} = \rho g \frac{Q}{b} S \equiv \rho g q S$$

(omega) Unit stream power

channel width

water discharge per unit width

What about sheer stress?

$$\omega = \rho g \frac{Q}{b} S \qquad Q = \underbrace{\Delta y}_{b} \underbrace{\Delta z}_{h} \bar{u}$$

$$\omega = \frac{\rho g h \cancel{b} \bar{u} S}{\cancel{b}} = \underbrace{\rho g h S}_{\text{depth-slope product}} \bar{u} \qquad \boxed{\omega = \tau_b \bar{u}}$$

Darcy-Weisbach

$$\tau_b = \rho \underset{\text{friction factor}}{C_f} \bar{u}^2 = \rho \underset{\text{shear velocity}}{u_*^2}$$

$$\bar{u} = \sqrt{\frac{\tau_b}{\rho C_f}}$$

$$\omega = \tau_b \bar{u} = \frac{\tau_b^{3/2}}{\sqrt{\rho C_f}} = \rho g \frac{Q}{b} S$$

Relating unit stream power to bedrock erosion

$$\omega = \rho g \frac{Q}{b} S$$

unit stream power

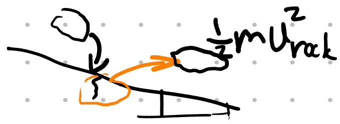
if uniform flow $\cancel{\bar{u}}$ $\cancel{q_s}$ $\dot{\varepsilon}$

(a) PE → KE (per time)

(b) PE → ME (per time)

Mechanical energy — heat (shear heating)

— breaking things!



guess

$$\boxed{\dot{\varepsilon} = k_\varepsilon \, \omega}$$

k_ε: Rock erodibility (& sediment w/ tools and cover)

$$\boxed{\dot{\varepsilon} = k_\varepsilon \rho g \frac{Q}{b} S}$$

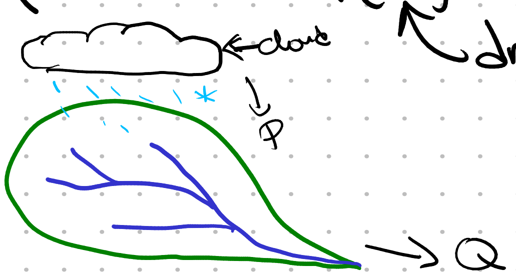
Can apply if Q, b, S are measured. :)

S: Digital elevation models

b: High-resolution imagery

Q: Difficult

$Q \longrightarrow f(A)$ ← drainage-basin area



$$Q = k_{AQ} A^{p_{AQ}}$$

power-law approximation (EMPIRICAL)

k_{AQ}: f(precip rate, p_{AQ}) ← P

$p_{AQ} \approx 0.6 \sim \underline{1.0}$

$Q = PA$

$$\dot{\varepsilon} = k_\varepsilon \rho g \frac{Q}{b} S$$

$$= \underset{\substack{\uparrow \\ f(\text{erodibility, sediment: tools or cover})}}{k_\varepsilon} \underset{\substack{\uparrow \\ f(P, P_{AQ})}}{k_{AQ}} \rho g \frac{A^{P_{AQ}}}{b} S$$

P = precip rate

Wolman & Miller: $b \propto Q^{1/2}$

$$b = k_{Qb} Q^{1/2}$$

$$\dot{\varepsilon} = \frac{k_\varepsilon k_{AQ}}{k_{Qb}} \rho g \frac{A^{P_{AQ}}}{Q^{1/2}} S \qquad Q^{1/2} = \left(k_{AQ} A^{P_{AQ}}\right)^{1/2}$$

$$Q^{1/2} = k_{AQ}^{1/2} A^{P_{AQ}/2}$$

$$\dot{\varepsilon} = \underbrace{\frac{k_\varepsilon k_{AQ}^{1/2}}{k_{Qb}} \rho g}_{} A^{P_{AQ}/2} S$$

$$\dot{\varepsilon} = K A^m S^n$$ ← in case of nonlinearity

$$\boxed{\dot{\varepsilon} = K A^m S^n}$$ ⇐ Stream-power law!

if $Q = PA$, then $P_{AQ} = 1$

Also: assume linearity: $\dot{\varepsilon} \propto \omega$

$$\dot{\varepsilon} = K A^{1/2} S^1 \quad \Leftarrow \quad m = 1/2, \; n = 1$$

$\dot{\varepsilon}$: $\left[\frac{m}{yr}\right]$, K: $\left[\frac{1}{yr}\right]$ rate constant, $A^{1/2}$: $[m]$, S^1: $[-]$

m: often 0.35–0.5 or so (no hard & fast rule)